Section II- Circular Motion

IIa – BANKING

Application of the Newton's Laws can explain why a turn is banked and why a car (with people) can go through a loop and not fall off at the top (or the people). The 1st Law states that an object moving at a constant velocity will continue moving in a straight line unless acted upon by a force. Even though the car may be traveling at a constant seep through the turn, if the direction of the speed is changing the velocity which it is a vector is a depended on the magnitude and direction. A change is velocity is acceleration and a force is required to accelerate an object. A force must be provided to turn the roller coaster car. This force is sometimes called the turning force and is provided by the track. More formally, since acceleration and force are related, this is called **Centurial Force or Centurial Acceleration.**

Note: The Force is always directed toward the center of the curve, or inward. It feels like you are being "forced" outward in a turn, but that is the effect of Newton's 1st Law. Your body wants to keep moving in a straight line, but a force is pushing you inward. It is the equal and opposite reaction of your body that makes you feel like a force is pushing you in an opposite direction. (Which Law is that?)

So it is only the **Centurial Force or Centurial Acceleration** that is real and used in the force diagrams to calculate the motion and response.

The formula for Centurial Acceleration is:

$$a_c = v^2 / r$$

And since F=ma; the Centurial Force is:

$$F_c = ma$$

Or

$$F_c = m\left(v^2 / r\right)$$

r is the radius of the curvature

Example: An air force jet is traveling at 200 m/s. After dropping some cluster bombs on some terrorists the pilot pulls up to avoid anti-aircraft fire in a circular path with a radius of 0.750 km.

(a) What is the acceleration of the airplane? Divide by G to show the answer in multiples of G.
(b) What is the force required if the plane has a mass of 25,000 kg?

Solution:

$$(a) a_c = \frac{v^2}{r}$$

$$a_c = \frac{(200 m/s)^2}{[0.750 km \times (1000 m/km)]}$$

$$a_c = 53.33 m/s^2$$

$$\frac{a_c}{g} = \frac{(53.33 m/s^2)}{(9.8 m/s^2)}$$

$$\frac{a_c}{g} = 5.44g$$

$$(b) F_c = 25{,}000 kg \times 53.33 m/s^2$$

$$F_c = 1{,}333{,}250 kg m/s^2$$

$$F_c = 1{,}333{,}250 N$$

(Remember 1 N = 1 kg m/s^2)

Examples Illustrating circular Motions Using the Roller Coaster

(1) Banked Curve

The car enters the 1st 180° turn with a velocity of 3.5 m/s (11.5 ft/sec). This turn has a radius of 20.32 cm (8.0"). What is the acceleration and force from the track on the car due to the turning? The car has a total mass of 324.5 grams (11.5 oz).

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(3.5m/s)^2}{[20.32cm \times (1m/100cm)]}$$

$$a_c = 60.3m/s^2$$

$$\frac{a_c}{g} = \frac{(60.3m/s^2)}{(9.8m/s^2)}$$

$$\frac{a_c}{g} = 6.15g$$

or

$$(a)a_c = \frac{v^2}{r}$$

$$a_c = \frac{11.5ft/\sec^2}{[8.0in \times (1ft/12in)]}$$

$$a_c = 198.4ft/\sec^2$$

$$\frac{a_c}{g} = \frac{(198.4ft/\sec^2)}{(32ft/\sec^2)}$$

$$\frac{a_c}{g} = 6.15g$$

$$F_c = ma_c$$

$$F_c = 324.5g(1kg/1000g)(60.3m/s^2)$$

$$F_c = 19.57kgm/s^2$$

$$F_c = 19.57N$$

or

$$F_c = 11.5oz(11b_m/16oz)(198.4ft/\sec^2)$$

$$F_c = 142.6lbm - ft/\sec^2$$

$$F_c = \frac{142.6lbm - ft/\sec^2}{32.2bm - ft/1bf - \sec^2}$$

$$F_c = 4.431bf$$

Did you get the same answer in both systems? (1 N = 0.2251bf)

But this still doesn't say why the curve should be banked. To illustrate this draw the force diagram on the car.

$$Fn_v = Fn\cos\theta$$
$$Fn_H = Fn\sin\theta$$
$$F_g = mg$$

The Force due to gravity must be overcome by the centripetal force.

$$F_c = Fn_H$$
$$m\left(\frac{v^2}{r}\right) = F_n \sin\theta$$
$$and$$
$$F_g = Fn_v$$
$$mg = Fn\cos\theta$$

One way to get rid of the unknown Fn is to divide the two equations:

$$\frac{m(v^2/r)}{mg} = \frac{F_n \sin\theta}{F_n \cos\theta}$$
$$\frac{v^2}{gr} = \frac{\sin\theta}{\cos\theta}$$
$$\frac{v^2}{gr} = \tan\theta$$

So now we can determine the optimum bank angle.

What is the maximum speed the roller coaster car must be traveling entering the loop to prevent falling off at the top? (Ignore friction) This problem can be treated as before, and exchange of kinetic energy into potential energy. Total energy at loop entry must equal the total energy at the top.

$$KE_{entry} + PE_{entry} = KE_{top} + PE_{tep}$$
$$\frac{1}{2}mV_2^1 + mgh_1 = \frac{1}{2}mV_2^2 + mgh_2$$

The mass cancels

Use the previous answer for the velocity at the top, V_2; and 25.4 cm(10”) loop height.

$$\frac{1}{22}V_1^2 = \frac{1}{2}V_2^2 + g(h_2 - h_1)$$
$$V_1 = \sqrt{\left[V_2^2 + 2g(h_1 - h_2)\right]}$$
$$V_1 = \sqrt{\left[(1.00m/s)^2 + 2\times 9.8m/s^2(25.4cm\times 1m/100cm)\right]}$$
$$V_1 = 2.45m/s$$

or

$$V_1 = \sqrt{\left[3.3ft/\sec^2 + 2\times 32.2ft/\sec^2(10in\times 1ft/12in)\right]}$$
$$V_1 = 8.0ft/\sec$$

Based on the answer from the problem velocity at the bottom of the hill (the entry into the loop) will the car fall?

What g-force will the occupants feel at the top of the loop using the predicted velocities of 3.5m/s (11.5ft/sec). Again this is a total energy problem. This time we know V_1 and want to calculate V_2.

$$\frac{1}{2}mV_2^1 + mgh_1 = \frac{1}{2}mV_2^2 + mgh_2$$
$$\frac{1}{2}V_2^2 = \frac{1}{2}V_2^2 + g(h_1 - h_2)$$
$$V_2 = \sqrt{\left[V_1^2 + 2g(h_1 - h_2)\right]}$$

$$V_2 = \sqrt{\left[(3.50m/s)^2 - 2\times 9.8m/s^2(25.4cm\times 1m/100cm)\right]}$$
$$V_2 = 2.70m/s$$

$$V_2 = \sqrt{\left[(11.5mft/\sec)^2 - 2\times 9.8m/s^2(10on\times 1ft/12in)\right]}$$
$$V_2 = 8.86ft/\sec$$

What g force does the car feel at the top of the loop? For this problem the forces are not in balance:

$$\sum F = ma$$
$$-F_a + F_g = ma$$

The negative sign is because the centripetal force acts in the opposite direction of gravity force.

$$-m\left(\frac{v^2}{r}\right) + mg = ma$$
$$-v\left(\frac{v^2}{r}\right) + g = a$$
$$\tan\theta = \frac{v^2}{gr}$$
$$\tan\theta = \frac{3.5m/s^2}{9.8m/s^2 \times 20.32cm(1m/100cm)}$$
$$\tan\theta = 6.15$$
$$\theta = 81°$$

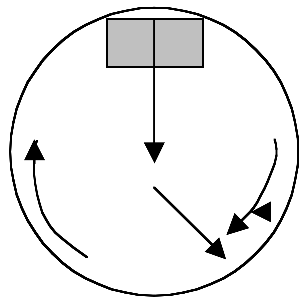
or

$$\tan\theta = \frac{11.5ft/\sec^2}{32.2ft/\sec^2 \times 8.0in(1ft/12in)}$$
$$\tan\theta = 6.15$$
$$\tan\theta = 81°$$

Of course should be the same no matter what units you are using. But note how the answer is related to the number of g's! This is a very interesting result and it applies to all types of motion. For example, the formula for banking angle of an airplane is the same tan _ = g.

The same balance forces are applied at the top of the loop:

To prevent the car from falling the centripetal force must be at least as great as the force due to gravity.



$$F_c = F_g$$

$$m\left(\frac{v^2}{r}\right) = mg$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{gr}$$

What is the minimum speed the car must be traveling at the top of the loop to prevent falling? Use the radius of curvature of 10.16cm (4").

$$V = \sqrt{gr}$$
$$V = \sqrt{9.8m/s^2 (10.16cm)(1/100cm)}$$
$$V = 1.00m/s$$

OR

$$V = \sqrt{32.2ft/\sec^2 (4in)(1ft/12in)}$$
$$V = 3.3ft/\sec$$

$$-(v^2/gr) + 1 = a/g$$
$$\frac{a.g}{} = 1 - (2.70m/s)^2 / [9.8m/s^2 / (10.16cm * 1m/100cm)]$$

$$\frac{a}{g} = -6.3g's$$

OR

$$\frac{a.}{g} = 1 - (8.86ft/\sec)^2 / [32.2ft/\sec^2 / (4in * 1ft/12in)]$$
$$\frac{a}{g} = -6.3g's$$

The negative sign means it is opposite to the gravity vector. The force is up!