Section I – Kinematics

A model roller coast is an ideal way to illustrate the physics of motion. The study of motion is called **kinematics** and is critical to engineering. Even though a roller coaster track twists and turns and has it ups and downs, the motion is considered motion is a plane. Also, the active force acting on the car is the force due to gravity. The forces due to friction are very small due to the precision design and manufacture of the model roller coaster. However, friction can be introduced as an advanced concept.

The math concepts required and practiced are:

(1) Algebra
(2) Trigonometry
(3) Vector manipulation

The physics illustrated by the model roller coaster and in this lesson plan are:

(1) Newton's Laws of Motion
(2) Instantaneous velocity and acceleration
(3) Vector decomposition of forces
(4) Circular motion

Definitions with Units

Acceleration**:** The rate of change of velocity
(ft/s/s –or- m/s/s) note: though it is convenient to say "feet-per-second-squared" the proper was is "feet-per-second -per second". It is okay to use the shorter version as long as the concept is understood.

Acceleration due to gravity, "g": On earth 32.2 ft/s/s or 9.81 m/s

Velocity: Distance divided by time. (ft/s –or- m/s)

Force: (lbf –or- N)

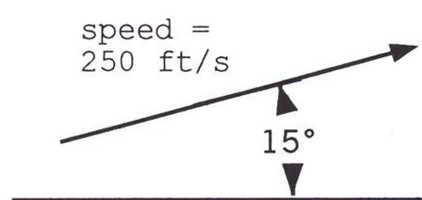
Mass: (lbm –or- kg)

Vector: A quantity that has both magnitude and direction.

ft – feet	m – meter	1 m = 3.281 ft
1bf – pound 'force'	N – Newton	1 N = 0.225 lbf
lbm - pound 'mass'	kg – kilogram	1 kg = 2.205 lbm
s – second		

Vector and Net Forces

A vector is a quantity, such as force, acceleration, and velocity where it is important to know the magnitude (size) and its direction. In plane motion, it is enough to give the magnitude and the angle. For example, the magnitude of velocity is speed, and its direction can be given as an angle relative to the horizontal. An airplane just after take-off has a speed of 250 gt/s and a climb angle of 15 degrees. The velocity is 250 ft/s @ 15°.



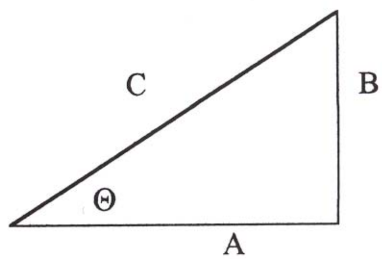
An important concept is the idea of **NET FORCE.** This means any group of forces can be combined through the use of vector addition and subtraction to result in a single, or net force. If this force replaces all of the other forces, it will cause the body to move in the exact same way. Also, the reverse is true, and single force can be broken replaced with any number of proper forces. In practice, forces will be broken down into two perpendicular forces. This can be done one of two ways:

(a) Since gravity is very important and only acts in one direction (down on earth) we can breakdown all forces into that act with or against gravity OR forces that act perpendicular to gravity. This means each force will have two components, on in the horizontal direction and one in the vertical direction. This is useful in free falling or projectile problems.

(b) At other times, motion in a fixed direction is important and every force is decomposed into a force along the direction of motion and a force perpendicular to the motion. Along the roller coaster track and perpendicular to the track would. In this case the force due to gravity will be broken down into 2 components, one along the track and another perpendicular to the track. We will see that this is a very useful way to describe the force of gravity since it will be the component of force along the track that controls the motion of the car while it's the force perpendicular to the track that must be counteracted to prevent the car from falling at the top of the loop.

A very helpful concept is that once the forces are broken down into the 2-perpendicular components each set of forces in a component can be added or subtracted separately and the laws of Motion apply to each component separately. For example, if all the forces are broken down as in method (I) we can sum all the forces that act with or against gravity and then predict the motion in the vertical direction. If any forces act perpendicular to gravity we can analyze those separately.
Before a few examples are shown we need to know a few trigonometric rules:

A, B, and C are the lengths of a triangle:



$$A^2 + B^2 = C^2$$

$$A = C\cos\theta$$

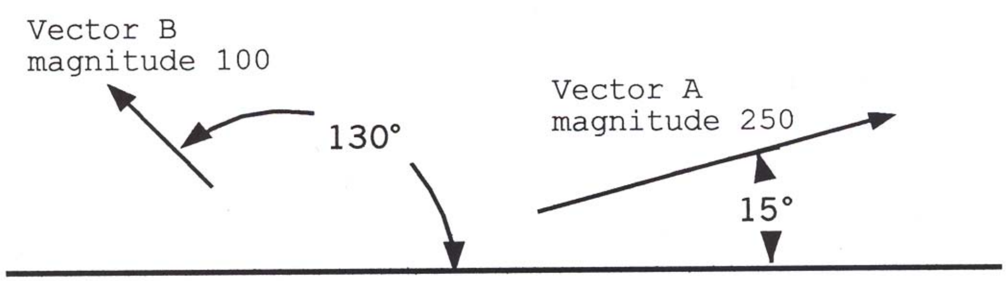
$$B = C\sin\theta$$

$$\theta = \arctan\left(\frac{B}{A}\right)$$

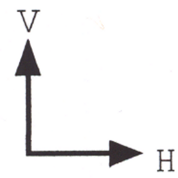
Vectors can be added and subtracted if they are in the same direction. We break apart a vector into two0compnents as above, so that each of the vectors breakdown into the same directions and can be added or subtracted.

Here is an example of vector math:

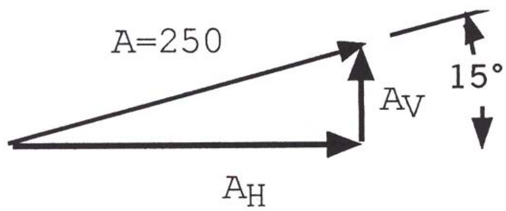
There are two vectors; it could be velocity, force, or and vector quantity. Combine these two vectors into a single vector.



First, choose the horizontal and vertical for the vector decomposition.



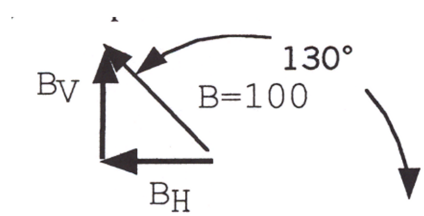
Next, break down vector A into horizontal and vertical components:



Substitute into the equations above:

$$A_v = 250\sin(15°) = 64.7$$
$$A_H = 250\cos((15°) = 241.5$$
$$Check: (64.7^2 + 241.5^2) = 250^2$$

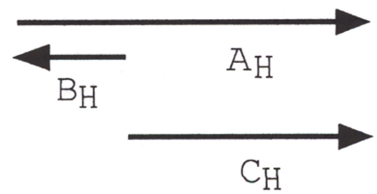
Now, decompose vector B:



$$B_V = 100\sin(130°) = 76.6$$
$$B_H = 100\cos(130°) = -64.3$$
$$Check: (76.6^2 + -64.3^2) = 100^2$$

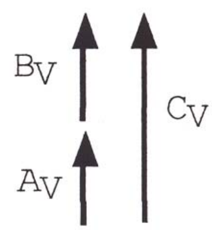
Now we can add the components of each vector that are in the same direction:

$C_H = A_H + B_H = 64.7 = 76.6$



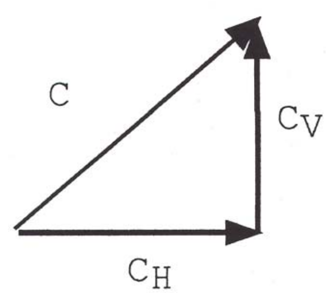
$C_H = 141.3$

$C_V = A_v + B_v = 241.5 + -64.3$



$C_V = 177.2$

Now the vertical components are combined to create vector C.



$$C^2 = \left(C_H^2 + C_V^2\right)$$

$$C = \sqrt{143.3^2 + 177.2^2}$$

$$C = 226.6$$

$$\theta = \arctan\left(\frac{C_V}{C_H}\right)$$

$$\theta = \arctan\left(\frac{177.2}{141.3}\right)$$

$$\theta = 51.4°$$

Thus the vector combination of A and B is: C=226.2 @ 51.4°.
So, if A and B were separate forces we could replace them with a single force that would have the exact same effect.